

OCT 31 1995

Name:

Student number

Computational Science 260

Midterm Exam

Nov. 23

Marks

1. Let P be "grass is green", Q be "ice is hot", and R be "the year has 400 days". Express the statement $(P \wedge \neg Q) \Rightarrow (\neg P \vee R)$ in English. Also, find the truth value of this expression. 10
2. A *group* is defined as a set, together with an operation which is associative, has an identity element, and an inverse for each element. For the following three cases, state whether or not the set is a group. In particular, state if the operation in question is associative, and indicate the identity element. Also, indicate how the inverse is normally expressed. 15
 1. \mathbb{N} with the operation $+$.
 2. \mathbb{Z} with the operation $+$.
 3. \mathbb{Z} with multiplication.
3. Is it possible that the union of two symmetric relation, both of which are defined on the same set A , is not symmetric? If your answer is negative, give an explanation in English. If your answer is positive, give an example. 8
4. Give a formal proof for $\exists x \neg \forall y P(x, y) \vdash \forall y \exists x \neg P(x, y)$. 15
5. Let $A = \{1, 2, 4\}$, and let $B = \{3, 4, 6\}$. Find $(A - B) \times (B - A)$. 10
6. Write Prolog predicates for the following definitions. Note that Prolog accepts the normal relational operators, such as $>$, $>=$, and so on. 16
 1. Write a predicate `wealthy(Age, Income)` which succeeds if the income exceeds 1000 times the age.
 2. Write a predicate `tax(Sale, Tax, Total)` which calculates a 14% tax on sales above \$1, and adds to Sales in order to find Total. If Sale is below \$1, the tax is zero.

Prolog

7. Use induction to show that $3n \leq n^2$ for all $n \geq 3$. Establish the base of induction, and the inductive step. 14

8. Let R and S be two predicates. Given $r(x,y)$ is stored as a fact in the data base if xRy , and that $s(x,y)$ similarly represents the relation S . Write a Prolog predicate `rcircs(X,Y)` which succeeds if (X,Y) are in the relation $R \circ S$. 12

Prolog

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————— The End —————